Section 3.7

Differentiating Compositions of Functions

Suppose there are two racers, call them f and g. We are told that racer f is traveling 20 m/s and racer g is traveling half again as fast. How far will racer g travel in 1 minute?

Theorem 3.7.1 (The Chain Rule)**.** Suppose $f(u)$ is differentiable at $u = g(x)$ and $g(x)$ is differentiable at x. Then $f(g(x))$ is differentiable at x and its derivative is given by

$$\frac{d}{dx}\big(f(g(x))\big) = f'(g(x))g'(x)$$

How to use the rule:

1. Identify inside and outside functions of a composition.
2. Differentiate the outside function and compose with the inside function.
3. Differentiate the inside function and multiply.

Example 3.7.2. Use the appropriate differentiation rules to find the derivatives of the following:

1. $y = (3x^2 + 7x)^{10}$

2. $y = e^{Kt}$

3. $f(x) = e^{\tan x}$

4. $w = \sin x^5$

5. $g(x) = \sqrt{x + \sqrt{x}}$

6. $h(x) = \tan(xe^x)$

7. $y = \sqrt[5]{x^4 + \cos 2x}$

A useful shortcut formula:

Theorem 3.7.3 (Chain Rule for Powers)**.** If g is differentiable for all x in its domain and p is a real number, then

$$\frac{d}{dx}\big(\big(g(x)\big)^p\big) = p\big(g(x)\big)^{p-1} g'(x).$$

Proof of Chain Rule: